Stress Testing and Bank Lending*

Joel Shapiro† and Jing Zeng‡

University of Oxford and Frankfurt School of Finance and Management

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Abstract

Bank stress tests are a major form of regulatory oversight implemented in the wake of the financial crisis. Banks respond to the strictness of the tests by changing their lending behavior. As regulators care about bank lending, this affects the design of the tests and creates a feedback loop. We analyze a model of this feedback effect when the regulator trades off the costs of reduced credit to the real economy versus the benefits of preventing default. Stress tests may be (1) lenient, in order to encourage lending in the future, or (2) tough, in order to reduce the risk of costly bank defaults. Reputational concerns on the part of the regulator drive how tough/lenient the test is. These equilibria may co-exist, and due to a strategic complementarity between the regulator's stress testing strategy and the bank's lending decision, may lead to fragility. We find that in some situations, surplus would be higher without stress testing and regulators may strategically delay stress tests.

Keywords: Bank regulation, stress tests, bank lending

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†Saïd Business School, University of Oxford, Park End Street, Oxford OX1 1HP. Email: Joel.Shapiro@sbs.ox.ac.uk

‡Frankfurt School of Finance and Management, J.Zeng@fs.de.

1 Introduction

Stress tests are a new policy tool for bank regulators that were first used in the recent financial crisis and have become regular exercises subsequent to the crisis. They are assessments of a bank's ability to withstand adverse shocks, and are generally accompanied by requirements intended to boost the capital of those banks who have been found to be at risk.

Naturally, bank behavior reacts to stress testing exercises. Acharya, Berger, and Roman (forthcoming) find that all banks that underwent the U.S. SCAP and CCAR tests reduced their risk by raising loan spreads and decreasing their commercial real estate credit and credit card loans activity.[1]

Regulators should take into account the reaction of banks when conducting the tests. One might posit that if regulators want to boost lending, they might make stress tests more lenient. Indeed, in the case of bank ratings, Agarwal et al. (2014) show that state level banking regulators rate banks more leniently than federal regulators due to concerns over the local economy and this may lead to more bank failures.

In this paper, we study the feedback effect between stress testing and bank lending. Banks may take excess risk or not lend enough to the real economy. Regulators react with either a lenient or tough approach. We demonstrate that this behavior may be self-fulfilling and result in coordination failures. A regulator may prefer uninformative stress tests or to strategically delay the test.

We examine a model in which there are two sequential stress testing exercises. For simplicity, only one bank is tested in each exercise. Banks choose how much effort to put in to originate a risky loan, with their outside option being to invest in a risk free asset. The regulator can observe the quality of the risky loan, and may require the bank to raise capital (which we denote as "failing" the stress test) or not. Therefore, stress tests in the model are about gathering information and boosting capital, rather than relaying information to the market. This is in line with the annual exercises during non-crisis times, where runs are an unlikely response to stress test results.[2]

In the model, there is uncertainty about whether a regulator is lenient. A lenient regulator conducts uninformative stress test exercises. A strategic regulator who may fail a bank trades off the cost of foregone credit[3] with the benefit of reducing costly default. After the first bank's stress test result, the second bank updates its beliefs about the preferences of the regulator, tries to originate a loan, and undergoes a stress test. Thus, the regulator's stress test for the first bank serve two roles: to possibly boost capital for the first bank and to signal the regulator's willingness to force banks to raise capital.

[1] Connolly (2017) and Calem, Correa, and Lee (2017) have similar findings.

[2] In the case where the bank is the same in both tests, this also resembles banking supervision (for a discussion of banking supervision, see Eisenbach et. al, 2017).

[3] Increased credit is positively associated with economic growth and income for the poor (both across countries and U.S. states, see Levine, 2011). Moskowitz & Garmaise (2006) provide causal evidence the social effects of credit allocation such as reduced crime.

Banks may take too much or too little risk from the regulator's point of view. On the one hand, a bank will take too much risk due to its limited downside. On the other hand, a bank may take too little risk when potential capital raising requirements extract rents from the bank's owners.[4] The banks' choices affect the leniency of the stress test and the leniency of the stress test affects the banks' choices.

The regulator faces a natural trade-off in dealing with the first bank:

First, the regulator may want to build a reputation for being lenient, which can increase lending by the second bank. Since lenient regulators prefer not to require banks to raise capital, there is an equilibrium where the regulator builds the perception that it is lenient by passing the bank in the first stress test. This is reminiscent of EU's 2016 stress test, where the pass/fail grading scheme was eliminated and only one bank was found to be undercapitalized.[5]

Second, the regulator may want to build a reputation for being tough, which can prevent future excess risk-taking. This leads to an equilibrium where it is tough in its first stress test: it fails banks that should pass with positive probability. Post-crisis, the U.S. has routinely been criticized for being too strict: its very adverse scenarios, not providing the model to banks, accompanying asset quality reviews, and qualitative reviews all feed into a stringent test.

Finally, there is one more type of equilibrium, where the regulator doesn't have reputation concerns and acts in accordance with its information.

Intriguingly, these equilibria can coexist, leading to a natural coordination failure. This occurs due to a strategic complementarity between the strategic regulator's choice of leniency at the first bank and the second bank's risk choice. The less likely the strategic regulator is to pass the first bank, the more risk the second bank takes when it observes a pass (as it believes the regulator is the lenient type). This prompts the strategic regulator to be tougher, and leads to a self-fulfilling equilibrium. This implies that the presence of stress tests may introduce fragility in the form of excess default or reduced lending to the real economy. The inefficiency in equilibrium choices may be sufficiently large that a regulator may prefer not to conduct stress tests or strategically delay them.

This naturally raises the issue of how a particular equilibrium may be chosen. One way might be if regulators can commit ex-ante to how to use information provided to them (as in games of Bayesian Persuasion). In practice, this might mean announcing stress test scenarios in advance or allowing banks to develop their own scenarios. The regulator might also take costly actions to commit by auditing bank data (e.g. asset quality reviews).

[4] Thakor (1996) provides evidence that the adoption of risk-based capital requirements under Basel I and the passage of FDICIA in 1991 led to banks substituting risky lending with Treasury investments potentially prolonging the economic downturn.

[5] That bank, Monte dei Paschi di Siena, had already failed the 2014 stress test and was well known by the market to be in distress.

In the model, uncertainty about the regulator's preferences plays a key role. Given that increased lending may come with risk to the economy, there is ample motivation to keep this information/intention private. Nevertheless, it is not the private information element that is important for our model; it is the fact that the market (including banks) is *uncertain* about what the preferences of the regulator are. This uncertainty may arise from the political process. Decisionmaking may be opaque, bureaucratic, or tied up in legislative bargaining.[6] Meanwhile, governments with a mandate to stimulate the economy may respond to lobbying from various interest groups or upcoming elections.[7]

There is little direct evidence of regulators behaving strategically during disclosure exercises, but much indirect evidence. The variance in stress test results to date seem to support the idea of regulatory discretion.[8] Beyond Agarwal et. al. (2014) cited above, Bird et al. (2015) show U.S. stress tests were lenient towards large banks and strict with poorly capitalized banks, affecting bank equity issuance and payout policy. The recent Libor scandal revealed that Paul Tucker, deputy governor of the Bank of England, made a statement to Barclays' CEO that was interpreted as a suggestion that the bank lower its Libor submissions.[9] Hoshi and Kashyap (2010) and Skinner (2008) discuss accounting rule changes that the government of Japan used to improve the appearance of its financial institutions during the country's crisis.[10]

Theoretical Literature

There are a few papers on reputation management by a regulator. Morrison and White (2013) argue that a regulator may choose to forbear when it knows that a bank is in distress, because liquidating the bank may lead to a poor reputation about the ability of the regulator to screen and trigger contagion in the banking system. Boot and Thakor (1993) also find that bank closure policy may be inefficient due to reputation management by the regulator, but this is due to the regulator being self-interested rather than being worried about social welfare consequences as in Morrison and White (2013). Shapiro and Skeie (2015) show that a regulator may use bailouts to stave off depositor runs and forbearance to stave off excess risk taking by banks. They define the type of the regulator as the regulator's cost of funding. We also have potential contagion through reputation as in these papers, but model the bank's lending decision and how it interacts with the choice of the regulator to force banks to raise capital. Furthermore, we define the regulator's type as whether it lenient (uninformative) or strategic.

[6] Shapiro and Skeie (2015) provide examples of related uncertainty around bailouts during the financial crisis.

[7] Thakor (2014) discusses the political economy of banking.

[8] The 2009 U.S. SCAP was widely perceived as a success (Goldstein and Sapra, 2014), with subsequent U.S. tests retaining credibility. European stress tests have varied in perceived quality (Schuermann, 2014) with the early versions so unsuccessful that Ireland and Spain hired independent private firms to conduct stress tests on their banks.

[9] The CEO of Barclays wrote notes at the time on his conversation with Tucker, who reportedly said, "It did not always need to be the case that [Barclays] appeared as high as [Barclays has] recently." This quote and a report on what happened appear in the *Financial Times* (B. Masters, G. Parker, and K. Burgess, Diamond Lets Loose Over Libor, *Financial Times*, July 3, 2012).

[10] Nevertheless, stress tests do contain significant information that is valued by markets (Flannery, Hirtle, and Kovner (2017) demonstrate this and survey recent evidence).

There are several recent theoretical papers on regulatory disclosure and stress tests. Goldstein and Sapra (2014) survey the stress test and disclosure literature to describe the costs and benefits of information provision. Prescott (2008) argues that more information disclosure by a bank regulator decreases the amount of information that the regulator can gather on banks. Bouvard, Chaigneau, and de Motta (2015) show that transparency is better in bad times and opacity is better in good times. Goldstein and Leitner (2018) find a similar result in a very different model where the regulator is concerned about risk sharing (the Hirshleifer effect) between banks. Williams (2017) looks at bank portfolio choice and liquidity in this context. Orlov, Zryumov, and Skrzypacz (2018) show that the optimal stress test will test banks sequentially. Faria-e-Castro, Martinez, and Philippon (2016) demonstrate that stress tests will be more informative when the regulator has a strong fiscal position (to stop runs). Compared to these papers, we don't allow the regulator to commit to a disclosure rule (as all of the papers except for Bouvard, Chaigneau, and de Motta (2015) do); *reputational incentives* drive the regulator's choices. In addition, we incorporate capital requirements as a key element of stress testing and focus on banks' endogenous choice of risk.

Our paper identifies the regulator's reputation concern as a source of fragility in the banking sector. In a different context, Ordonez (2013, 2017) show banks' reputation concerns, which provides discipline to keep banks from taking excessive risk, can lead to fragility and a crisis of confidence in the market.

2 The model

We consider a model with five types of risk-neutral agents: the regulator, banks, borrowers, existing debtholders and capital providers. In the model, the regulator will conduct two stress tests. To keep things simple, we only allow for each stress test to examine one bank. The bank examined may be the same in both stress tests, or different. We will refer to them as different banks for clarity. We assume that the regulator has a discount factor $\delta \geq 0$ for the payoffs from the second bank, where δ may be larger than 1 (as, e.g., in Laffont and Tirole, 1993). The discount factor captures the relative importance of the future of the banking sector for the regulator. For simplicity, we do not allow for discounting within a bank.

2.1 Banks and rollovers

For each bank t, where $t = \{1, 2\}$, there are four stages:

1. The bank exerts effort to originate a risky loan. If it does not find an appropriate loan, it invests in the safe asset;

2. A risky loan may be good or bad quality. The regulator conducts the stress test by observing the

quality of bank t's asset, and decides whether to pass or to fail the bank. In the case of failure, the regulator requires the bank to raise capital;

3. The bank rolls over or liquidates its maturing debt;

4. The payoff of the bank realizes.

Before the start of the game, the bank has raised one unit of debt (or uninsured deposits). At stage 1, the bank has access to a safe investment opportunity which returns $R_f > 1$ at stage 4. With probability e_t, the bank identifies an appropriate risky loan at stage 1. Originating this loan incurs a quadratic screening cost of $\frac{1}{2}ke_t^2$. In order to focus on the reputation building incentives of the regulator when conducting the first bank's stress test, we make the simplifying assumption that the first bank has the opportunity to extend a risky loan with fixed probability e_1.[11] For the second bank, we assume that the bank can choose its costly loan origination effort to improve the chance that it identifies a risky loan.

If the bank extends a risky loan, the loan quality q_t can be good (g) or bad (b), where the prior probability that the loan is good is denoted by α. The loan can be liquidated at stage 3 to generate 1. If not liquidated, a good loan ($q_t = g$) repays R with probability 1 at stage 4, whereas a bad loan ($q_t = b$) repays R with probability $1 - d$ and 0 otherwise at stage 4. We make the following assumption about the loan returns.

Assumption 1. *(i)* $\alpha R > R_f$*; (ii)* $(1 - d)R > 1$.

Part (i) of Assumption 1 ensures that the bank extends a risky loan whenever the opportunity arises.[12] This is because the bank's owners receive at least αR from the risky loan, which repays R with certainty if it is good (with probability α). If the loan is bad (with probability $1 - \alpha$), the bank's owners may not receive the value of the loan because it may have to raise capital (as required by the regulator during a stress test, see Section 2.2), which incurs dilution costs. Assumption 1 therefore provides a sufficient condition. Part (ii) of Assumption 1 implies that liquidation is inefficient at stage 3.

We make the following parameter restriction on the marginal cost of effort k to ensure a unique interior solution to the bank's effort problem in equilibrium.

Assumption 2. $k > [\alpha + (1 - \alpha)(1 - d)]R - R_f$.

We assume that it is observable to all market participants whether the bank has made a safe investment or a risky one. As we will discuss below, the regulator will learn about the credit quality of the risky investment.

[11] Allowing endogenous loan origination effort by the first bank does not alter the reputation building incentives we demonstrate in Section 4.

[12] This claim is formally shown in the proof of Proposition 1.

The bank's debt matures at stage 3 and has an exogenous promised repayment of 1.[13] Since the bank's asset only matures at stage 4, the bank must rollover its debt at stage 4. Based on whether the bank has invested in a risky loan at stage 1 and the information revealed by the stress test results at stage 2, the bank rolls over its debt with an endogenously determined promised repayment at stage 4 of $\tilde{R} \in [1, R]$. For simplicity, we assume that the bank has all the bargaining power, so that $\tilde{R}$ is such that the expected payoff to the debtholders is equal to 1. Assumption 1 implies that the bank is always solvent, and thus refinancing is always feasible and efficient at stage 3.

2.2 The regulator and stress testing

The regulator conducts the stress test by observing the quality q_t of the bank's risky loan at stage 2, and then decides whether to require the bank to raise capital. We will henceforth refer to the regulatory action of requiring the bank to raise capital as "fail", and not requiring the bank to raise capital as "pass". The regulator's objective function is to maximize the expected value of the bank minus the social cost of bank defaults and recapitalizations.

The stress test in the model therefore is not about conveying information to the market about the health of the banks. The test provides the regulator with information on the bank's health, which the regulator uses by requiring recapitalizations. Nevertheless, the stress test accompanied by the recapitalizations[14] does convey information to the market. This information is about the private information of the regulator (defined below). In the model, the second bank reacts to this information, forming the basis of the reputation mechanism.

If a bank fails the stress test, we assume that the bank is required to raise 1 unit of capital kept in costless storage with zero net return so that the bank with a risky loan will not default at stage 4 even if its borrower does not repay.[15] There is a capital provider who can fund the bank. We assume that the capital provider has some bargaining power due to the scarcity of capital, enabling it to capture a fraction β of the expected surplus of the bank. Raising capital thus results in a (private) dilution cost for the bank's owners. The banking literature generally views equity capital raising as costly for banks (for a discussion see Diamond (2017)). We model this cost as dilution due to bargaining power from capital providers, which fits our scenario of a public requirement by a regulator, though other mechanisms that impose a cost on the bank when trying to shore up capital are possible.[16]

[13] While the promised repayment can be greater than 1, the renegotiation-proof repayment is equal to 1 when the bank has all the bargaining power. This is because the liquidation value of the debt is equal to 1.

[14] The stress test results themselves are cheap talk in the model, but the recapitalizations incur costs (and benefits) for the regulator, making costly signals possible.

[15] We assume capital earns zero net return for simplicity. The results do not change if capital is reinvested in the safe investment with a return $R_f > 1$.

[16] For example, the bank may be forced to sell assets at fire-sale prices. This is a loss in value for the bank. And those who

The regulator can be of two types, the strategic type or the lenient type. The lenient type always passes the bank, whereas the strategic type trades off the social benefits and costs associated with capital when deciding whether to fail a bank, as detailed just below. The regulator knows its own type, but during the stress testing of bank t (where $t = \{1, 2\}$), the market (the existing debtholders of bank t, capital providers and the owners of bank t) is uncertain about the regulator's type. The market has an ex ante belief that, with probability $1 - z_t$, the regulator is strategic. With probability z_t, the regulator is believed to be a lenient type. In our model, z_1 is the probability that nature determines that the regulator is a lenient type. The term z_2 is the updated belief held by the market that the regulator is a lenient type after bank 1's stress test.

The social benefit of capital is to eliminate the cost to society of a bank default at stage 4. Specifically, if a bank operates without recapitalization and the borrower repays 0 at stage 4, the bank defaults and a social cost to society D is incurred. The cost of bank default may represent the loss of value from future intermediation the bank may perform, the cost to resolve the bank, or the cost of contagion.

The strategic regulator has a social cost of capital of C from recapitalization. The social cost of capital here reflects the opportunity cost of the capital used, i.e. the positive externality generated by alternative projects the capital providers could invest in. Note that the social cost of capital is often thought of as the positive value lost from the bank not investing in other projects, not from the capital provider's foregone investments. Nevertheless, stress tests have generally been accompanied by dividend restrictions, compensation restrictions, and seeking outside funding. Plantin (2014) similarly models the forgone cost of outside investment opportunities due to equity capital raising. Stein's (2012) patient investors are like our capital provider in that they forgo investments to provide capital to the bank, albeit in return for assets.

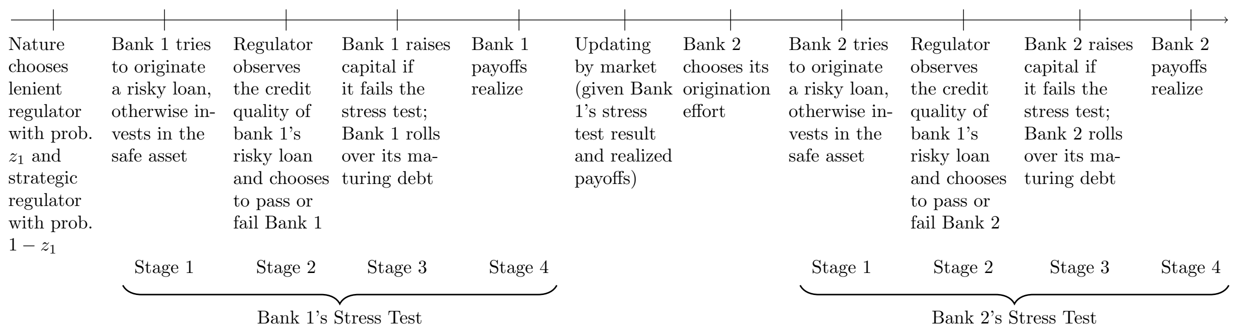
Therefore when a bank is recapitalized, the social cost is incurred, but the expected social benefit depends on expectations about the likelihood that the bank's risky loan produces a low return. We make the following assumption about the relative social costs and benefits of capital.

Assumption 3. *(i)* $dD > C > 0$*; (ii)* $[\alpha + (1-\alpha)(1-d)]\,R - R_f > (1-\alpha)dD$.

Part (i) of this assumption states that the strategic regulator finds that it is beneficial to recapitalize a bank whose risky loan is known to be bad. This puts the strategic regulator in conflict with the lenient regulator and drives reputational incentives. Part (ii) of this assumption states that the risky loan produces a higher expected value than the risk free investment even after taking into account the social cost associated with a potential bank default. This assumption ensures that it is socially desirable that the bank invests in the risky asset with positive probability.

are purchasing the assets are distorting their investment decisions, as in our model. Hanson, Kashyap, and Stein (2011) discuss this effect and review the literature on fire sales.

Figure 1: Time line of events



The lenient type, on the other hand, always passes the bank. The stress test conducted by the lenient type regulator is therefore uninformative. Agents may view this type as not conducting "serious" stress test exercises. The behavior of the lenient type regulator can be microfounded by a high social cost of capital relative to the social benefit of capital.[17]

2.3 Summary of timing

The regulator conducts stress testing of the two banks sequentially. The timing is illustrated in Figure 1.

The stress testing of bank 2 begins after the stress test of bank 1 finishes, so that the beliefs about the type of the regulator will be updated depending only on the result of bank 1's stress test and the realized payoff of bank 1.

We assume that the probabilities that the risky loan opportunity is good at the second bank is independent of whether the risky loan opportunity is good at the first bank, and that the type of the regulator is independent from the quality of the banks' risky loans. Furthermore, the regulator's type remains the same when testing both banks.

We use the concept of perfect Bayesian equilibrium. We use the concept of D1 refinement of Cho and Kreps (1987) to refine off-equilibrium beliefs where possible.[18]

3 The second bank's stress test

We begin the analysis of the model by using backward induction, and characterize the equilibrium for the second bank. We first characterize the strategic regulator's stress test strategy at bank 2 at stage 2, taking

[17]Specifically, if the lenient type regulator faces a high social cost of capital $\bar{C}$ and/or a low cost of bank default $\underline{D}$, then passing the bank with certainty is indeed the unique equilibrium strategy if $\bar{C} \geq d\underline{D} + \delta\frac{1}{2}k$, i.e. that the high cost of capital for the lenient regulator exceeds the maximum benefit of capital over two banks. This includes a direct benefit at the first bank by the eliminating the expected cost of a bank default dD and a potential indirect benefit due to a reduced screening cost for the second bank, if failing the first bank increases the market's perception that the regulator has a high cost of capital. The maximum benefit at the second bank due to a reduced screening cost is no greater than $\frac{1}{2}k$.

[18]This is a commonly used refinement, see, for example, Nachman and Noe (1994) and DeMarzo and Duffie (1999).

as given the bank's loan origination effort in stage 0 and investment decision in stage 1. We then analyze the bank's investment decision and screening effort.

If the bank makes a safe investment at stage 1, it is clear that the bank will not default and therefore requires no capital at stage 2. The bank will be able to roll over its maturing debt at stage 3 with a promised repayment of $\tilde{R} = 1$. In this section, we will focus on describing the equilibrium stress test outcome given that the bank extends a risky loan at stage 1 to a good borrower.

Because the game ends after the second bank's stress testing results, the regulator has no reputational incentives. The stress test strategy of the strategic regulator at stage 2 depends on the quality of the bank's risky loan $q_2 \in \{g, b\}$. Specifically, the strategic regulator passes the bank only if the loan is good, as implied by Assumption 3. Table 1 depicts the regulator's equilibrium stress testing strategy.

Table 1: Equilibrium stress testing outcome of the second bank

	Strategic regulator	Lenient regulator
$q_2 = g$	Pass	Pass
$q_2 = b$	Fail	Pass

At stage 3, given the stress test result, the bank raises capital if it fails the stress test, and rolls over its maturing debt. Let α_2^p and α_2^f denote the market's posterior belief that that the bank's risky loan is good, given that the bank passes (henceforth denoted by superscript p) and fails (henceforth denoted by superscript f) the stress test, respectively. Given the regulator's stress testing strategy depicted in Table 1, the market's posterior beliefs are given by

$$\alpha_2^p = \frac{\alpha}{1 - (1 - z_2)(1 - \alpha)}, \qquad \alpha_2^f = 0$$

The promised repayment when the bank rolls over its maturing debt at stage 3 depends on the stress test results and the market's posterior belief about the bank's type. First, only a bank with with a bad risky loan fails the stress test. It is then required by the regulator to raise 1 unit of capital at stage 3. The capital allows the bank to roll over its maturing debt with a promised repayment of $\tilde{R}_2^f = 1$. Because of the capital providers' bargaining power, the bank's expected payoff captures a fraction $1 - \beta$ of the surplus in this case and is given by $(1 - \beta)[(1 - d)R - 1]$. Second, if a bank passes the stress test, it rolls over its maturing debt at stage 3 by promising a repayment $\tilde{R}_2^p(\alpha_2^p)$, such that

$$[\alpha_2^p + (1 - \alpha_2^p)(1 - d)]\, \tilde{R}_2^p(\alpha_2^p) = 1 \tag{1}$$

We can now analyze the bank's screening effort at stage 0. At stage 0, the bank takes as given the rollover repayments $\tilde{R}_2^p$ and $\tilde{R}_2^f$ in case it passes and fails the stress test. The bank chooses its screening effort to maximize the expected payoff to its owners net of effort cost. That is,

$$\max_{e_2} \quad \underbrace{e_2 \left(\overbrace{[\alpha + (1-\alpha) z_2 (1-d)] \left(R - \tilde{R}_2^p\right)}^{\text{pass}} + \overbrace{(1-\alpha)(1-z_2)(1-\beta)[(1-d)R - 1]}^{\text{fail}} \right)}_{s=g} + \underbrace{(1-e_2)(R_f - 1)}_{s=b} - \underbrace{\frac{1}{2} k e_2^2}_{\text{effort cost}} \tag{2}$$

The first term of Eq. 2 represents the expected payoff to the bank's owners when it originates a risky loan, with probability e_2. Subsequently, if the bank passes the stress test, it rolls over its debt with a promised repayment of $\tilde{R}_2^p$. The bank's owners thus receives the residual payoff $R - \tilde{R}_2^p$ if the loan is good (with probability α) or if the loan is bad but repays R when facing a high-cost regulator (with probability $(1-\alpha)z_2(1-d)$). If the loan is bad and the regulator has a low cost of capital, the bank fails the stress test. In this case, the expected payoff to the owners of the bank is given by $(1-\beta)[(1-d)R-1]$. The second term of Eq. 2 represents the expected payoff to the bank's owners when it invests in the safe asset (with probability $1-e_2$). In this case, the expected payoff to the owners of the bank is $(R_f - 1)$. Finally, the bank incurs the cost of loan origination effort.

Let us now characterize the equilibrium for bank 2 in the following proposition.

Proposition 1. *For the second bank, there exists a unique equilibrium in which the bank expends loan origination effort $e_2^*(z_2)$ at stage 1. If the bank extends a risky loan, at stage 2, the lenient type regulator passes the bank with certainty and the strategic regulator passes the bank with certainty if and only if the bank's loan is good.*

The bank's loan origination effort in equilibrium $e_2^*(z_2)$ is increasing in the market's belief that the regulator has a high cost of capital, z_2, where $e_2^*(z_2)$ is given by

$$\left(\underbrace{[\alpha + (1-\alpha)(1-d)]R - R_f}_{NPV\ effect} - \underbrace{(1-\alpha)(1-z_2)\beta[(1-d)R-1]}_{dilution\ cost} \right) - k e_2^*(z_2) = 0 \tag{3}$$

Notice that, given the regulator's stress testing strategy in equilibrium, the bank's choice of loan origination effort trades off the expected marginal gain from extending a risky loan (the first term of Eq. 3) against the marginal cost of screening effort (the second term of Eq. 3). The expected marginal gain from extending a risky loan has two components. On the one hand, the bank benefits from extending a risky loan because it produces a higher expected return than the safe investment (*NPV effect*). On the other hand, the bank faces a dilution cost whenever it is required to raise capital, because the capital providers require a higher rate of return (*dilution cost*). Since the bank only faces the possibility of failing the stress test and thus having to raise capital if it extends a risky loan, the bank's expected marginal gain from extending a risky loan also reflects the potential dilution cost of recapitalization.

Importantly, the regulator's reputation of being the lenient type, z_2 increases the second bank's loan origination effort in equilibrium, $e_2^*(z_2)$. This is because, the lenient type regulator does not require the bank to raise capital even if the bank's risky loan is bad (Table 1). As a result, the bank's incentive to originate a risky loan is greater when the bank expects that the regulator is of the lenient type, because the bank's owners receive the higher expected return from the risky loan without having to incur a dilution cost due to recapitalization.

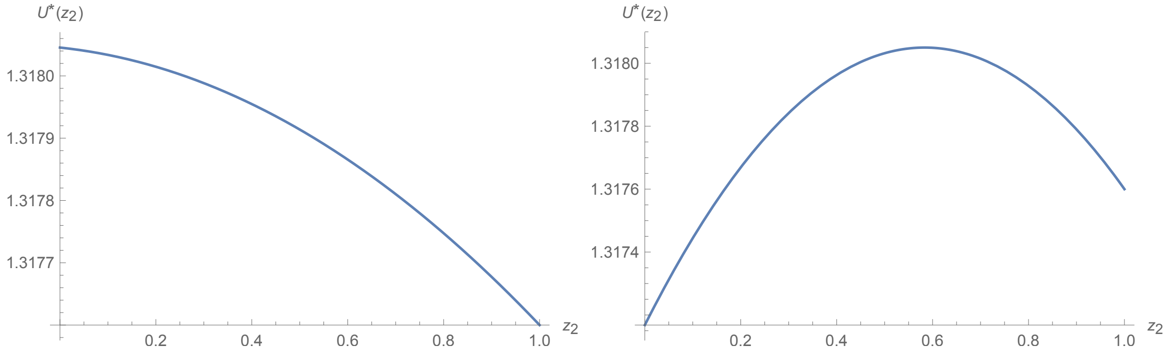
Since the regulator's reputation z_2 determines the bank's screening incentives in equilibrium, we now turn to understanding how the regulator's reputation affects its surplus (i.e. welfare). Let $U(e_2)$ denote the strategic regulator's expected surplus from the second bank given the bank's screening effort e_2. We can express the expected surplus as follows:

$$U(e_2) = \gamma e_2 \left(\left[\alpha + (1-\alpha)(1-d) \right] R - 1 - (1-\alpha)C \right) + (1-\gamma e_2)(R_f - 1) - \frac{1}{2} k e_2^2 \tag{4}$$

In the above expressions, the first term represents the expected surplus when the bank extends a risky loan, the second term represents the surplus when the bank invests in the safe asset, and the last term represents the cost of the bank's screening effort. Notice that, when the bank extends a risky loan, the strategic regulator internalizes the social cost of a potential bank default, which is equal to C, the social cost of bank recapitalization.

Crucially, because the regulator internalizes the social cost of a bank default, whereas the bank's does not, the bank's private choice of screening effort characterized in Proposition 1 generally differs from the socially optimal level. Moreover, the wedge between the two depends on the regulator's reputation, z_2. If the bank expects the regulator to be strategic (z_2 low), it expects a tough stress test and a high probability of having to raise capital when it extends a risky loan. In this case, the bank will be too conservative, and expend too little loan origination effort, resulting in too little lending. If the bank perceives the regulator to be of the lenient type (z_2 high), however, it expects a lenient stress test and a low probability of having to raise capital even if it extends a risky loan. In this case, the bank does not internalize the social cost of its risky investment and exerts too much screening effort, resulting in excessive risk taking. This implies that the expected surplus of the strategic regulator will be increasing in the regulator's reputation z_2 for z_2 low, and decreasing for z_2 high. In particular, this leads to a strictly interior level of reputation $\hat{z}$ that maximizes

Figure 2: The expected surplus for the second bank in equilibrium, given a strategic regulator. The parameter values used in this plot are $\alpha = 0.4$, $\gamma = 0.1$, $R = 5$, $R_f = 1.3$, $d = 0.5$, $D = 1.2$, $C = 0.5$, $\delta = 0.9$ and $k = 1$. The left panel represents the case where $\beta = 0.3 < \underline{\beta} = \frac{1}{3}$, and the right panel represents the case where $\beta = 0.8 > \underline{\beta}$.



the expected surplus of the strategic regulator. This result is formally stated in the following proposition and depicted in Figure 2. Let $U^*(z_2) = U(e_2^*(z_2))$ denote the expected surplus from the second bank for the strategic regulator in equilibrium.

Proposition 2. *The expected surplus from the second bank for the strategic regulator in equilibrium, $U^*(z_2)$, is strictly increasing in the probability that the regulator has a high cost of capital z_2 for $z_2 < \hat{z}$, and strictly decreasing in z_2 for $z_2 \geq \hat{z}$. $\hat{z} \in [0, 1)$ is defined by*

$$\hat{z} = \begin{cases} 0, & \text{if } \beta \leq \underline{\beta} \\ e^{*-1}\left(\frac{\gamma}{k}\left([\alpha + (1-\alpha)(1-d)]R - R_f - (1-\alpha)C\right) - k\right), & \text{if } \beta > \underline{\beta} \end{cases} \tag{5}$$

where $\underline{\beta}$ is defined by

$$\underline{\beta}\left[(1-d)R - 1\right] = C \tag{6}$$

Recall that β reflects the surplus extracted from the bank by the capital provider. The cutoff $\underline{\beta}$ defines the situation where the bank's private cost of capital for the bank is equal to the social cost of capital for the strategic regulator. Both the private cost of capital and the social cost of capital reflect the opportunity cost of foregone loans by the capital provider. The private cost of capital may additionally reflect the bargaining power of the bank vis-a-vis capital providers or a monitoring/information premium demanded by the capital provider. The social cost of capital may additionally reflect the positive externalities forgone by the capital provider channeling its funds to the bank. Therefore, depending on these measures, the private cost of capital may be higher or lower than the social cost of capital.

Proposition 5 thus states that, the expected surplus of the second bank in equilibrium is hump-shaped in its reputation of being the lenient type if the social cost of capital is lower than the private cost of capital, for reasons discussed above. However, if the social cost of capital is greater than the private cost of capital, then the expected surplus for the second bank in equilibrium for the strategic regulator is always decreasing in its reputation of being the lenient type. This is because when the private cost of capital is low, the bank always tends to over-invest the risky loans. Therefore the more the bank expects that the regulator is of the lenient type who does not require the bank to raise capital, the more risk the bank takes and the lower is the expected surplus.

4 The first bank's stress test

In this section, we analyze the equilibrium stress testing strategy of the regulator for the first bank, given the equilibrium at the second bank.

Let us consider the incentives of the strategic regulator to pass the first bank. Let G_{q_1} denote the net gain of passing the bank relative to failing the bank, given the quality of the bank's risky loan $q_1 \in \{g, b\}$.

$$\begin{aligned} G_g &= \underbrace{C}_{\text{bank 1 surplus effect}} + \underbrace{\delta \left[U^*(z_2^R) - U^*(z_2^f) \right]}_{\text{reputation effect}} \\ G_b &= \underbrace{(C - dD)}_{\text{bank 1 surplus effect}} + \underbrace{\delta \left[(1-d)U^*(z_2^R) + dU^*(z_2^0) - U^*(z_2^f) \right]}_{\text{reputation effect}} \end{aligned} \tag{7}$$

where the first term represents the net gain in terms of the expected surplus at the first bank, and the second term represents the reputation concern in terms of the expected surplus at the second bank. The terms z_2^R and z_2^0 are the posterior beliefs held by the market about the probability that the regulator is of the lenient type, given that the first bank passes the stress test and given the realized payoff of the first bank, R and 0, respectively. The term z_2^f is the posterior beliefs held by the market about the probability that the regulator is of the lenient type, given that the first bank fails the stress test.[19]

We first show that there is an equilibrium in which the strategic regulator's stress testing strategy for the first bank is identical to its strategy for the second bank.

Proposition 3. *The equilibrium stress testing strategy of the strategic regulator for the second bank described in Proposition 1 is an equilibrium strategy for the first bank. That is, if the bank extends a risky loan, at stage 2, the strategic regulator passes the bank if and only if the risky loan is good.*

[19]Note that since only a strategic regulator fails the first bank in equilibrium, the posterior belief $z_2^f = 0$ and does not depend on the realized payoff of the first bank.

There exist thresholds $\bar{\delta}_g(z_1)$ *and* $\bar{\delta}_b(z_1)$*, such that the necessary and sufficient conditions for this equilibrium to exist is* $\delta < \min\{\bar{\delta}_g(z_1), \bar{\delta}_b(z_1)\}$*.*

For certain parameters, the equilibrium stress testing strategy of the regulator for the first bank is the same as the strategy for the second bank, and is illustrated in Table 1. In such an equilibrium, the posterior probabilities that the regulator has a high cost of capital given that a pass result of the first bank's stress test and the realized payoff of the first bank are given by

$$z_2^R = \frac{[\alpha + (1-\alpha)(1-d)]\, z_1}{[\alpha + (1-\alpha)(1-d)]\, z_1 + \alpha(1-z_1)}, \qquad z_2^0 = 1, \qquad z_2^f = 0 \tag{8}$$

In particular, since only a lenient type regulator would pass a bad bank, the market associates a belief of $z_2^0 = 1$ with the low payoff 0 of a bank who passes the stress test; since only a strategic regulator would fail a bank, the market associates a belief of $z_2^f = 0$ with the failure of the first bank. If the first bank passes the stress test and realizes a high payoff R, the market updates its belief to $z_2^R > z_1$, reflecting the fact that a lenient type regulator is more likely to pass a good bank than a strategic regulator.

The stress testing strategy described in Proposition 3 is an equilibrium if and only if $G_g \geq 0 \geq G_b$. The stress testing strategy of the regulator at the first bank is affected by its reputation concerns, in addition to the expected surplus from the first bank (Eq. 7). While the effect through the first bank's expected surplus is always positive for a bank with a good risky loan and negative for a bank with a bad risky loan, the reputation effect may be positive or negative in equilibrium. This is because, as discussed in Proposition 2, the regulator's surplus is non-monotonic in its reputation. On the one hand, a reputation of being strategic implies a tough stress test, which can induce the second bank to be excessively conservative and reduce lending. On the other hand, a reputation of being the lenient type can result in excessive risk taking by the second bank and excessive lending.

Proposition 3 states that an equilibrium in which the regulator adopts the same stress testing strategy for both banks exists whenever the reputation effect does not outweigh the direct effect through the expected surplus at the first bank. There can be two cases. If the prior probability that the regulator has a high cost of capital (z_1) is low, the reputation effect of passing the first bank is positive, and the equilibrium exists if the reputation effect (δ) is not so large that the strategic regulator wishes to pass a bank with a risky loan of low credit quality. If the prior probability that the regulator has a high cost of capital (z_1) is high, the reputation effect of passing the first bank is negative, and the equilibrium exists if and reputation effect (δ) is not so large that the strategic regulator wishes to fail a bank with a risky loan of high credit quality.

For the 2009 U.S. SCAP stress test, one might argue that the discount factor was not high; i.e. the regulator was concerned more about the present than building their reputation. Moreover, given that the

Table 2: Equilibrium stress testing outcome of the first bank when the strategic regulator wants to build reputation to incentivize lending by the second bank

	Strategic regulator	Lenient regulator
$q_1 = g$	Pass	Pass
$q_1 = b$	Pass with probability $\pi_b^* > 0$	Pass

U.S. had explicitly committed to recapitalize banks that the exercise found to be distressed, the term z_1 was very low. We demonstrate that in this case, the stress test runs smoothly - the regulator conveys its information perfectly and recapitalization only take place for failed banks.

Proposition 3 shows that if the regulator's reputation concern δ is sufficiently small, the regulator's stress testing strategy for the first bank is identical to that for the second bank. However, we will show in the following sections that two other equilibria exist due to the regulator's reputation building incentives.

4.1 Reputation building to incentivize lending

Consider the incentives for the strategic regulator to pass the first bank with a bad risky loan. If the strategic regulator fails the first bank and recapitalizes it, the regulator reveals the fact that it is strategic to the market. The second bank then faces a strong incentive to exert less loan origination effort and to invest more in the safe asset, in order to avoid failing the stress test. If the strategic regulator passes the first bank, however, it pools with the lenient type regulator, increasing the incentive for the second bank to exert loan origination effort and engage in risky lending. If the benefit of increased lending by the second bank is sufficiently large, the regulator may want to pass the first bank even when the risky loan of the first bank is bad, in order to gain a reputation of leniency.

In the following proposition, we demonstrate that reputation building incentives to encourage lending by the second bank lead to another equilibrium for the stress testing of the first bank besides that of Proposition 3.

Proposition 4. *There is an equilibrium in which the strategic regulator passes the first bank with positive probability even when the first bank's risky loan is bad. Specifically, the regulator passes the bank with certainty if the loan is good, and passes the bank with positive probability $\pi_b^* > 0$ if the loan is bad.*

There exist $\underline{\delta}_b(z_1)$, such that the necessary and sufficient condition for this equilibrium to exist is $\beta > \underline{\beta}$ and $\delta \geq \underline{\delta}_b(z_1)$.

Table 2 depicts the stress testing outcome of the first bank in the equilibrium described in Proposition 4. Compared to the equilibrium in Proposition 3, the strategic regulator now follows a mixed strategy when facing a bank with a risky loan of low credit quality and passes this bank with positive probability. In equilibrium, the posterior probabilities that the regulator has a high cost of capital given a pass result of the first bank's stress test and the realized payoff of the first bank are given by

$$z_2^R = \frac{[\alpha + (1-\alpha)(1-d)]z_1}{[\alpha + (1-\alpha)(1-d)]z_1 + [\alpha + (1-\alpha)(1-d)\pi_b^*](1-z_1)}, \quad z_2^0 = \frac{z_1}{z_1 + \pi_b^*(1-z_1)}, \quad z_2^f = 0 \qquad (9)$$

In Proposition 3, the strategic regulator recapitalizes the bank with a bad risky loan, which maximizes the expected surplus from the bank. Passing the bank with a bad risky loan is therefore costly as it will incur a default cost with positive probability. However, by now passing the bank, the strategic regulator is able to increase the perception that it is of the lenient type, since the lenient type regulator always passes the bank. This is useful to the strategic regulator because it increases the incentives for the second bank to originate risky loans. In other words, the regulator enjoys a positive reputation effect from passing the first bank with a bad risky loan.

Proposition 4 identifies two necessary and sufficient conditions for an equilibrium with reputation building to incentivize lending to exist. First, the private cost of capital (β) must be sufficiently high, stemming from, e.g. fire-sale cost or the bargaining power of capital providers. As a result, the bank may under-expend loan origination effort when it expects recapitalization by the regulator, implying that there is a positive reputation effect of passing the first bank in equilibrium. Second, the reputation concern (δ) of the regulator must be sufficiently high, so that the regulator's reputational benefits outweigh the short-term efficiency loss when passing the first bank with a risky loan of low credit quality.

While the initial European stress tests performed poorly (e.g. passing Irish banks and Dexia), one might argue that during crisis times, the main focus was preventing runs - and without a fiscal backstop it was hard to maintain credibility (Faria-e-Castro, Martinez, and Philippon, 2016). We argue that in normal times, a stress test may be lenient to incentivize banks to lend to the real economy. This may explain the 2016 EU stress test, which eliminated the pass/fail criteria, reduced the number of banks stress tested by about half, used less adverse scenarios than the U.S. and UK, and only singled out one bank as undercapitalized - Monti dei Paschi di Siena, which had failed the previous (2014) stress test and was well known to be in distress.

4.2 Reputation building to reduce excessive risk-taking

Consider the incentives for the strategic regulator to fail the first bank with a good risky loan. If the regulator fails the first bank and recapitalizes it, the regulator reveals the fact that it is strategic to the market. The second bank then faces a strong incentive to invest in the safe asset, in order to avoid failing the stress test. If the regulator passes the first bank, however, it pools with the lenient type regulator. The second bank thus expects a lenient stress test and tends to invest excessively in the risky asset, because it does not internalize

Table 3: Equilibrium stress testing outcome of first bank when the strategic regulator wants to build reputation to reduce excessive risk-taking by the second bank

	Strategic regulator	Lenient regulator
$q_1 = g$	Pass with probability $\pi_g^* < 1$	Pass
$q_1 = b$	Fail	Pass

the social cost of a potential bank default. If the concerns about excessive risk-taking by the second bank are sufficiently large, the strategic regulator may want to fail the first bank with a risky loan even when it is good, in order to reveal to the market its willingness to fail a bank during the stress test.

In the following proposition, we demonstrate that the reputation building incentives to reduce excessive risk-taking by the second bank leads to an equilibrium for the stress testing of the first bank different than that of Proposition 3 and 4.

Proposition 5. *There is an equilibrium in which the strategic regulator fails the first bank with positive probability even when the first bank's risky loan is good. Specifically, when the first bank extends a risky loan, strategic regulator fails the bank with certainty if the loan is bad, and fails the bank with positive probability if the loan is good.*

There exists $\bar{\beta} > \underline{\beta}$, such that the necessary and sufficient condition for this equilibrium to exist is $\beta < \bar{\beta}$ and $\delta \geq \bar{\delta}_g(1)$.

Let $\pi_g^* \in (0,1)$ denote the probability that the strategic regulator passes the bank a good risky loan in equilibrium. Table 3 depicts the stree testing outcome of the first bank in the equilibrium described in Proposition 4. Compared to Proposition 3, the strategic regulator now follows a mixed strategy when facing a bank with a good risky loan and fails the bank with positive probability. In equilibrium, the posterior probabilities that the regulator is of the lenient type given a pass result of the first bank's stress test and the realized payoff of the first bank are given by

$$z_2^R = \frac{[\alpha + (1-\alpha)(1-d)]z_1}{[\alpha + (1-\alpha)(1-d)]z_1 + \alpha\pi_g^*(1-z_1)}, \quad z_2^0 = 1, \quad z_2^f = 0 \tag{10}$$

In Proposition 3, the strategic regulator passes the first bank with a good risky loan. Failing the bank in this case would result in a costly recapitalization of the first bank with no benefit, since the good loan will not default. However, by now failing this bank, the strategic regulator is able to reveal to the market its willingness to recapitalize a bank, and thus reduce the second bank's incentive to engage in excessive risk taking.

Proposition 5 identifies two necessary and sufficient conditions for an equilibrium with reputation building to reduce excessive risk-taking to exist. First, the private cost of capital (β) must be sufficiently low. The second bank takes excessive risk because its cost of being recapitalized β is low. This implies a negative reputation effect of passing the first bank in equilibrium, because the inefficiency stemming from excessive risk-taking dominates the potential inefficiency associated with too little lending. Second, the reputation concern (δ) of the regulator must be sufficiently high, so that the incentive for the strategic regulator to reduce excessive risk-taking by the second bank is sufficiently strong.

U.S. stress tests have generally been regarded as much more strict than European ones. First, the Federal Reserve performs the stress test itself on data provided by the banks (and does not provide the model to the banks), whereas in Europe, it has been the case that the banks themselves perform the test. Second, the U.S. stress tests have regularly been accompanied by Asset Quality Reviews, whereas this has been infrequent for European stress tests. Third, one of the most feared elements of the U.S. stress tests has been the fact that there is a qualitative element that can (and has been) used to fail banks. In line with our results above, the fact that U.S. stress tests have been institutionalized as occuring on a yearly basis implies that reputation concerns are important. Furthermore, a swifter recovery from the crisis means that capital raising for banks is likely to be easier in the U.S.

5 Equilibrium summary and multiplicity

We now show formally that the three equilibria discussed in the previous sections are the only possible equilibria with an informative stress test. Figure 3 plots the regions of the parameter space (δ, z_1) for which each equilibrium exists.

Proposition 6. *There exists an equilibrium for the first bank's stress test, and the only equilibria are*

A: an equilibrium without reputation building, as described by Proposition 3,

B: an equilibrium with reputation building to incentivize lending, as described by Proposition 4, and

C: an equilibrium with reputation building to reduce excessive risk-taking, as described by Proposition 5.

Recall that the reputation effect of the regulator's stress testing strategy for the first bank is given by the second term of Eq. 7. Given that the lenient type regulator passes the bank, if the strategic regulator fails the first bank, it is revealed to be strategic ($z_2^f = 0$) and have a strong willingness to recapitalize the bank if the bank's risky investment is of bad quality. Subsequently, the second bank behaves too conservatively. In contrast, if the strategic regulator passes the first bank, it is pooled with the lenient type regulator who also passes the first bank. Subsequently, the second bank perceives the regulator's willingness to recapitalize the bank as weak, and invests excessively in extending risky loans. In equilibrium, the posterior probability that the regulator is of the lenient type, given that the first bank passes the stress test is given by

$$z_2^R = \frac{[\alpha + (1-\alpha)(1-d)]z_1}{[\alpha + (1-\alpha)(1-d)]z_1 + [\alpha\pi_g + (1-\alpha)(1-d)\pi_b](1-z_1)}, \quad z_2^0 = \frac{z_1}{z_1 + \pi_b(1-z_1)} \tag{11}$$

where π_g and π_b denote the regulator's probability of passing the first bank, given that the first bank's risky loan is good and bad, respectively.

Propositions 3–5 imply that there exist parameter ranges such that more than one equilibrium exists. This is because the reputation concern of the regulator may feature a strategic complementarity between the regulator's stress testing strategy and the market's belief updating process. For certain parameters, the regulator's reputation concern is self-fulfilling.

On the one hand, the regulator's stress testing strategy and the market's belief updating process are strategic substitutes for (π_g, π_b) such that $z_2^R, z_2^0 < \hat{z}$. This range is where this expected surplus is increasing in z_2^R and z_2^0 (this is summarized in Proposition 2). Here, the market realizes that the strategic regulator's surplus is increasing when it is perceived to be the lenient type. If the market conjectures that the strategic regulator adopts a tougher stress testing strategy (lower π_g or π_b), the market infers that the regulator who passed the first bank is more likely to be the lenient type (higher z_2^R and z_2^0). Consequently, the second bank takes more risk after a pass result for the first bank, resulting in *higher* expected surplus $U_L(e_2^*)$. In turn, this increases the net gain for the strategic regulator from passing the first bank and the regulator should adopt a more lenient stress testing strategy.

On the other hand, the strategic regulator's stress testing strategy and the market's belief updating process are strategic complements for (π_g, π_b) such that z_2^R, $z_2^0 \geq \hat{z}_L$. This range is where the regulator's surplus is decreasing in z_2^R and z_2^0. Here, the market realizes that the strategic regulator's surplus is decreasing when it is perceived to be the lenient type. If the market conjectures that the strategic regulator adopts a tougher stress test strategy (lower π_g or π_b), the market infers that the regulator who passes the first bank is more likely to be strategic (higher z_2^R and z_2^0). Consequently, the second bank increases its risk-taking, resulting in *lower* expected surplus $U_L(e_2^*)$. In turn, this further decreases the net gain for the strategic regulator from passing the first bank, justifying a tougher testing strategy. It is indeed this strategy complementarity that leads to equilibrium multiplicity.

5.1 Welfare comparison of equilibria

We now analyze the welfare implication of the regulator's reputation concern during stress testing. In particular, for parameter values where multiple equilibria coexist, we compare the expected surplus of the regulator from the two banks.

Proposition 7. • *Whenever an equilibrium with reputation building to incentivize lending (Equilibrium B) coexists with another type of equilibrium, the expected surplus of the strategic regulator for the two banks is strictly higher in Equilibrium B.*

- *Whenever an equilibrium with reputation building to reduce excessive risk-taking (Equilibrium C) coexists with another type of equilibrium, the expected surplus of the strategic regulator for the two banks is strictly lower in Equilibrium C.*

This result demonstrates that when there are multiple equilibria, we can rank the equilibria in terms of welfare. When the strategic regulator chooses the probability with which to pass a bank with credit quality q_1, it takes as constant the beliefs of the market about its choice. A first best solution wouldn't take this as constant, creating a wedge between the optimal choice and the regulator's choice. The first best solution when there are multiple equilibria is to choose as lenient a stress test as possible. The reason for this is that, as described above, multiple equilibria occur when the strategic regulator's second period surplus is decreasing in its posterior reputation. Its posterior reputation is decreasing in the levels of leniency π_g and π_b (this can be seen in Eq. 11). Therefore the first best solution would choose as much leniency as possible. Therefore, the most lenient type of equilibrium (Equilibrium B) yields the highest welfare and the toughest equilibrium (Equilibrium C) yields the lowest welfare.

This suggests that despite heavy criticism, lenient stress tests in Europe may have been optimal given the constraints.

This equilibrium welfare ranking is sensitive to the setup of the model. For example, allowing the lenient type regulator to fail a bank or endogenizing the choice of the first bank's effort could change the ordering.

Nevertheless, the benefits from leniency summarized above mean that the regulator might be even better off by not conducting stress tests for the first bank. Notice that there may exist an equilibrium in which both types of regulator pass the first bank with certainty. This is equivalent to an economy where the regulator does not conduct stress tests for the first bank.

Proposition 8. *There exists $\tilde{\delta}_b(z_1) \geq \underline{\delta}_b(z_1)$, such that an equilibrium in which the regulator passes the first bank with certainty exists if and only if $\delta \geq \tilde{\delta}_b(z_1)$. When such an equilibrium exists, it produces the highest expected surplus for the strategic regulator among all equilibria.*

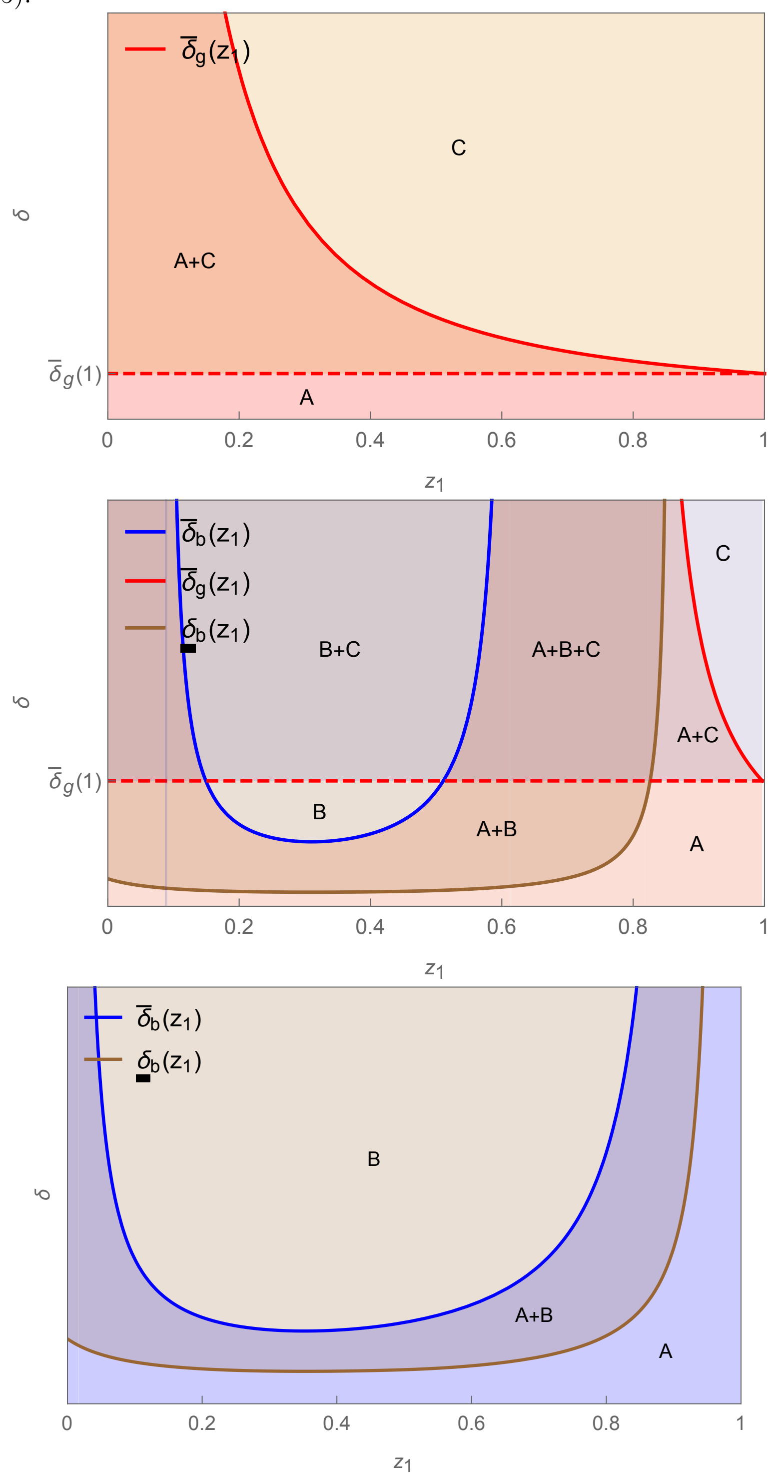
This proposition shows that no stress test for the first bank yields the highest welfare among all equilibria, whenever it can be supported as an equilibrium outcome. However, because of equilibrium multiplicity, the economy may be trapped in a less efficient equilibrium. Therefore if the regulator could strategically commit to not doing a stress test at the first bank or delaying this stress test, there could be substantial welfare gains. The timing of European stress tests has been quite irregular compared with the annual U.S. exercises (they were conducted in 2010, 2011, 2014, 2016, and 2018). Delay in this situation may be a way of committing to lenience.

The fact that we have three potentially coexisting equilibria raises the issue of how a particular equilibrium may be chosen. Commitment by the regulator in an ex-ante stage would facilitate this. Of course, in a crisis, committing to future actions may not be feasible. The regulator has access to several policy variables that might prove useful as commitment devices. Committing to how signals from banks are used is standard in the Bayesian Persuasion literature, but requires substantial independence from political pressure and processes that are well defined. A more practical alternative is committing to stress test scenarios. Stress test scenarios can be more or less lenient, given the effect desired. Asset quality reviews also commit more resources and reveal more information about bank positions.

6 Conclusion

Stress tests have been incorporated recently into the regulatory toolkit. The tests provide assessments of bank risk in adverse scenarios. Regulators respond to negative assessments by requiring banks to raise capital. However, regulators have incentives to be tough by asking even some safe banks to raise capital or to be lenient by allowing some risky banks to get by without raising capital. These incentives are driven by the weight the regulator places on lending in the economy versus stability. Banks respond to the leniency of the stress test by altering their lending policies. We demonstrate that in equilibrium, regulators may be tough and discourage lending or lenient and encourage lending. These equilibria can be self-fulfilling and the regulator may get trapped in one of them.

Figure 3: Parameter space for the existence of different equilibrium. The parameter values used in this plot are the same as for Figure 2 except β. The top panel represents the case where $\beta = 0.3 < \underline{\beta} = \frac{1}{3}$, the middle panel represents the case where $\beta = 0.65 \in [\underline{\beta}, \bar{\beta}]$, and the bottom panel represents the case where $\beta > \bar{\beta} = \frac{2}{3}$. The letters represent different equilibrium outcomes: A represents the equilibrium without reputation building (Proposition 3), B represents the equilibrium with reputation building to incentivize lending (Proposition 4) and C represents the equilibrium with reputation building to reduce excessive risk-taking (Proposition 5).



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7 Proofs

7.1 Proof of Proposition 1

The regulator's stress testing strategy at stage 2 is as described in the discussion in Section 3. The bank's effort choice $e_2^*(z_2)$ is given by Eq. 2. The first order condition that characterizes the bank's effort choice is

$$[\alpha + (1-\alpha)z_2(1-d)]\left(R - \tilde{R}_2^p\right) + (1-\alpha)(1-z_2)(1-\beta)[(1-d)R-1] - (R_f - 1) - ke_2 = 0$$

After substituting in $\tilde{R}_2^p(\alpha_2^p)$ given by Eq. 1, the bank's optimal screening choice in equilibrium satisfies Eq. 3.

We can now show that a unique equilibrium exists. This follows because the LHS of Eq. 3 is strictly decreasing in e_2, is strictly positive for $e_2 = 0$, and is strictly negative for $e_2 = 1$ (by Assumption 2).

It follows that the equilibrium screening effect is increasing in the regulator's reputation z_2

$$\frac{\partial e_2^*(z_2)}{\partial z_2} = \frac{\beta(1-\alpha)[(1-d)R-1]}{k} > 0 \tag{12}$$

Let us now verify that the bank indeed prefers to extend a risky loan whenever the opportunity arises. Suppose the equilibrium is as described in Proposition 1. Then given a good signal, the bank's expected payoff from extending the risky loan is given by

$$\begin{aligned}&[\alpha + (1-\alpha)z_2(1-d)]\left(R - \tilde{R}_2^p\right) + (1-\alpha)(1-z_2)(1-\beta)[(1-d)R-1]\\ =&[\alpha + (1-\alpha)(1-d)]R - 1 - \beta(1-\alpha)[(1-d)R-1] > R_f - 1\end{aligned} \tag{13}$$

where the last inequality follows from Assumption 1. Therefore in the equilibrium described in Proposition 1, the bank extends a risky loan whenever the opportunity arises.

7.2 Proof of Proposition 2

$U(e_2)$ is given by Eq. 4. Notice that $U(e_2)$ is increasing in e_2 if and only if $e_2 \leq \hat{e}$, where $\hat{e} \in (0,1)$ is defined by

$$\hat{e} = \max\left\{0, \frac{1}{k}\left([\alpha + (1-\alpha)(1-d)]R - R_f - (1-\alpha)C\right)\right\} \tag{14}$$

Recall that the bank's equilibrium screening effort $e_2^*(z_2)$ is increasing in z_2, where are given by

$$\begin{aligned} e_2^*(0) &= \frac{1}{k}\left(\left[\alpha+(1-\alpha)(1-d)\right]R - R_f - (1-\alpha)\beta[(1-d)R-1]\right) \\ e_2^*(1) &= \frac{1}{k}\left(\left[\alpha+(1-\alpha)(1-d)\right]R - R_f\right) \end{aligned} \tag{15}$$

Notice that $e_2^*(1) > \hat{e}$ and $e_2^*(0) < \hat{e}$ if and only if $\beta > \underline{\beta}$, where $\underline{\beta}$ is defined by Eq. 6.

Therefore there exists $\hat{z} \in [0, 1)$, such that $U(e^*(z_2))$ is increasing in z_2 if and only if $z_2 \leq \hat{z}$, where $\hat{z}$ is given by Eq. 5. That is, if $\beta \leq \underline{\beta}$, $\hat{z} = 0$, and if $\beta > \underline{\beta}$,

$$e_2^*(\hat{z}) = \hat{e} \tag{16}$$

7.3 Proof of Proposition 3

Before we proceed to prove this proposition, it is useful to define parameter values such that $U^*(0) > U^*(1)$.

$$\begin{aligned} U^*(1) - U^*(0) &= \left[e_1^*(1) - e_2^*(0)\right]\left([\alpha+(1-\alpha)(1-d)]R - R_f - (1-\alpha)C\right) - \frac{1}{2}k\left[\left(e_2^*(1)\right)^2 - \left(e_2^*(0)\right)^2\right] \\ &= \frac{1}{k}(1-\alpha)\beta[(1-d)R-1]\left([\alpha+(1-\alpha)(1-d)]R - R_f - (1-\alpha)C\right) \\ &\quad - \frac{1}{2}\frac{1}{k}(1-\alpha)\beta[(1-d)R-1]\left(2\left([\alpha+(1-\alpha)(1-d)]R - R_f\right) - (1-\alpha)\beta[(1-d)R-1]\right) \end{aligned}$$

Therefore $U^*(0) > U^*(1)$ if and only if $\beta < \bar{\beta}$, where $\bar{\beta} > \underline{\beta}$ is defined by

$$\bar{\beta}[(1-d)R-1] = 2C \tag{17}$$

If $\beta > \bar{\beta}$, Proposition 2 then implies that $U^*(0) \leq U^*(z_2)$ for all $z_2 > 0$. If $\beta < \bar{\beta}$, Proposition 2 implies that, there exists $\tilde{z} \in (\hat{z}, 1)$, such that $U^*(0) > U^*(z_2)$ if and only if $z_2 > \tilde{z}$, where $\tilde{z}$ is defined by

$$\tilde{z} = \begin{cases} = 0, & \text{if } \beta < \underline{\beta}, \\ \text{is defined by } U^*(0) = U^*(\tilde{z}), & \text{if } \beta \in [\underline{\beta}, \bar{\beta}] \\ = 1, & \text{if } \beta > \bar{\beta} \end{cases} \tag{18}$$

Let us now derive the conditions for the equilibrium described in Proposition 3 to exist. Conjecture an equilibrium in which the strategic regulator passes the bank with a risky loan if and only if its credit quality is high. In this equilibrium, the market's posterior beliefs about the regulator's type given the first bank's stress test results are given by Eq. 8. This is an equilibrium if and only if $G_g > 0 > G_b$.

Consider first G_g. There exists $\tilde{z}_g$ such that $U^*(\frac{[\alpha+(1-\alpha)(1-d)]z_1}{[\alpha+(1-\alpha)(1-d)]z_1+\alpha(1-z_1)}) > U^*(0)$ if and only if $z_1 < \tilde{z}_g$, where $\tilde{z}_g < \tilde{z}$ is defined by

$$\tilde{z}_g = \begin{cases} = 0, & \text{if } \beta < \underline{\beta} \\ \text{is defined by } U_L^*(0) = U_L^*(\frac{[\alpha+(1-\alpha)(1-d)]\tilde{z}_g}{[\alpha+(1-\alpha)(1-d)]\tilde{z}_g+\alpha(1-\tilde{z}_g)}), & \text{if } \beta \in [\underline{\beta}, \bar{\beta}] \\ = 1, & \text{if } \beta > \bar{\beta} \end{cases} \tag{19}$$

This implies that $G_g > 0$ for all $z_1 \leq \tilde{z}_g$.

For all $z_1 > \tilde{z}_g$, there exists $\delta_g(z_1)$, such that $G_{g,L} > 0$ if and only if $\delta < \bar{\delta}_g(z_1)$, where $\bar{\delta}_g(z_1)$ is defined by

$$C + \bar{\delta}_g(z_1)\left[U^*(\frac{[\alpha+(1-\alpha)(1-d)]z_1}{[\alpha+(1-\alpha)(1-d)]z_1+\alpha(1-z_1)}) - U^*(0)\right] = 0 \tag{20}$$

Notice that $\bar{\delta}_g(z_1)$ as defined above exists for $z_1 > \tilde{z}_g$ and is decreasing in z_1. Moreover, $\bar{\delta}_g(z_1) \to \infty$ as z_1 approaches $\tilde{z}_g$ from above. For completeness, let us define $\bar{\delta}_g(z_1) = \infty$ for $z_1 \leq \tilde{z}_g$.

Consider next G_b. There exists $\tilde{z}_b$ such that $(1-d)U^*(\frac{[\alpha+(1-\alpha)(1-d)]z_1}{[\alpha+(1-\alpha)(1-d)]z_1+\alpha(1-z_1)} + dU^*(1) < U^*(0)$ if and only if $z_1 > \tilde{z}_b$, where $\tilde{z}_b \leq \tilde{z}_g$ is defined by

$$\tilde{z}_b = \begin{cases} = 0, & \text{if } \beta < \underline{\beta} \\ \text{is defined by } U^*(0) = (1-d)U^*(\frac{[\alpha+(1-\alpha)(1-d)]\tilde{z}_b}{[\alpha+(1-\alpha)(1-d)]\tilde{z}_b+\alpha(1-\tilde{z}_b)}) + dU^*(1), & \text{if } \beta < \bar{\beta} \\ = 1, & \text{if } \beta \geq \bar{\beta} \end{cases} \tag{21}$$

This implies that $G_b < 0$ for all $z_1 \geq \tilde{z}_b$.

For all $z_1 < \tilde{z}_b$, then there exists $\bar{\delta}_b(z_1)$, such that $G_{b,L} < 0$ if and only if $\delta < \bar{\delta}_b(z_1)$, where $\bar{\delta}_b(z_1)$ is defined by

$$C - dD + \bar{\delta}_b(z_1)\left[(1-d)U^*(\frac{[\alpha+(1-\alpha)(1-d)]z_1}{[\alpha+(1-\alpha)(1-d)]z_1+\alpha(1-z_1)}) + dU^*(1) - U^*(0)\right] = 0 \tag{22}$$

Notice that $\bar{\delta}_b(z_1)$ as defined above exists for $z_1 < \tilde{z}_b$. $\bar{\delta}_b(z_1)$ is decreasing in z_1 if and only if $z_1 \geq \hat{z}_g$, where $\hat{z}_g$ is defined by

$$\frac{[\alpha+(1-\alpha)(1-d)]\hat{z}_g}{[\alpha+(1-\alpha)(1-d)]\hat{z}_g+\alpha(1-\hat{z}_g)} = \hat{z}_L \tag{23}$$

Moreover, $\bar{\delta}_b(z_1) \to \infty$ as z_1 approaches $\tilde{z}_b$ from below. For completeness, let us define $\bar{\delta}_b(z_1) = \infty$ for $z_1 \geq \tilde{z}_b$.

To summarize, the equilibrium described in Proposition 3 exists if $\delta \leq \min\{\bar{\delta}_g(z_1), \bar{\delta}_b(z_1)\}$, where $\bar{\delta}_g(z_1)$ and $\bar{\delta}_b(z_1)$ are defined by Eq. 20 and Eq. 22, respectively.

7.4 Proof of Proposition 4

This proof follows similar logical steps as the proof of Proposition 3. Consider first the strategic regulator facing a bank with a bad risky loan. In this case, the regulator's incentive to pass the bank is given by G_b, where z_2^R, z_2^0 and z_2^f are given by Eq. 9. The regulator passes the bank with probability $\pi_b^* \in (0, 1]$ if and only if $G_b \geq 0$. This is the case if and only if $\delta > \underline{\delta}_b(z_1)$, where $\underline{\delta}_b(z_1)$ is defined by

$$
\begin{aligned}
C - dD + \underline{\delta}_b(z_1) \max_{\pi_b \in [0,1]} \left[(1-d)U^*(z_2^R) + dU^*(z_2^0) - U^*(0)\right] = 0 \\
\text{where} \quad z_2^R &= \frac{[\alpha + (1-\alpha)(1-d)]z_1}{[\alpha + (1-\alpha)(1-d)]z_1 + [\alpha + (1-\alpha)(1-d)\pi_b](1-z_1)} \\
z_2^0 &= \frac{z_1}{z_1 + \pi_b(1-z_1)}
\end{aligned} \tag{24}
$$

Notice that $\underline{\delta}_b(z_1)$ as defined above only exists for $\beta > \underline{\beta}$ and for z_1 such that

$$
\max_{\pi_b \in [0,1]} \left[(1-d)U^*(z_2^R) + dU^*(z_2^0) - U^*(0)\right] < 0 \tag{25}
$$

For completeness, let us define $\underline{\delta}_b(z_1) = \infty$ for all z_1 such that $\max_{\pi_b \in [0,1]} \left[(1-d)U^*(z_2^R) + dU^*(z_2^0) - U^*(0)\right] \geq 0$.

Consider next the strategic regulator facing a bank with a good risky loan. Since $z_2^R < z_2^0$, $G_b \geq 0$ implies that $U^*(z_2^R) > U^*(0)$. This then implies that $G_g > 0$ for z_2^R and z_2^0 define by Eq. 9.

To summarize, an equilibrium described in Proposition 4 exists if and only if $\beta > \underline{\beta}$ and $\delta \geq \underline{\delta}_b(z_1)$, where β is defined by Eq. 6 and $\underline{\delta}_b(z_1)$ is defined by Eq. 24.

7.5 Proof of Proposition 5

This proof follows similar logical steps as the proof of Proposition 3. Consider first the strategic regulator facing a bank with a good risky loan. In this case, the regulator's incentive to pass the bank is given by G_g, where z_2^R, z_2^0 and z_2^f are given by Eq. 10. The regulator fails the bank with probability $\pi_g^* \in [0, 1)$ if and only if $G_{g,L} \leq 0$. Notice that this implies that $\beta < \bar{\beta}$ and $z_1 > \tilde{z}_g$. Further, $G_{g,L}$ is strictly increasing in π_g for all (π_g, π_b) such that $G_{g,L} \leq 0$. Therefore there exists $\pi_g^* < 1$ if and only if $G_g \leq 0$ for $\pi_g = 0$. This is the case if and only if $\beta < \bar{\beta}$ and $\delta \geq \bar{\delta}_g(1)$, where $\bar{\delta}_g(z_1)$ is defined by Eq. 20.

Consider next the regulator facing a bank with a bad risky loan. For $\beta < \bar{\beta}$, $U^*(1) < U_L^*(0)$. It then follows that $G_g \leq 0$ implies that $G_b < 0$.

To summarize, the equilibrium with in informative stress test described in Proposition 3 exists if and only if $\beta < \bar{\beta}$ and $\delta \geq \bar{\delta}_g(1)$, where $\bar{\delta}_g(z_1)$ is defined by Eq. 20.

7.6 Proof of Proposition 6

Follows immediately from the proofs of Proposition 3–5.

7.7 Proof of Proposition 7

Let $V^A(z_1)$, $V^B(z_1)$ and $V^C(z_1)$ denote the expected surplus for the strategic regulator for the two banks given that the first bank extends a risky loan, in the equilibria described in Propositions 3, 4 and 5, respectively, assuming the equilibria exist. Let $\underline{V}$ denote the expected surplus for the strategic regulator for the two banks given that the first bank extends a risky loan, given that the regulator fails the first bank with certainty (i.e. $\pi_g = \pi_b = 0$). $\underline{V}$ is given by

$$\underline{V}(z_1) = [\alpha + (1-\alpha)(1-d)]R - C + \delta U^*(0) \tag{26}$$

In an equilibrium in which the regulator's stress testing strategy is (π_g^*, π_b^*), the expected surplus for the strategic regulator for the two banks given that the first bank extends a risky loan is given by

$$V(z_1) = \underline{V}(z_1) + \left[\alpha\pi_g^* G_g + (1-\alpha)\pi_b^* G_b\right] \tag{27}$$

where G_{q_1} is given by Eq. 7. Let $G_{q_1}^A$, $G_{q_1}^B$ and $G_{q_1}^C$ denote the value of G_{q_1} in the equilibria described in Propositions 3, 4 and 5 respectively, assuming the equilibria exist.

By the proofs of Proposition 3–5, G_{q_1} satisfies the following properties in each equilibrium:

Equilibrium C	$G_g^C \leq 0$	$G_b^C < 0$
Equilibrium A	$G_g^A > 0$	$G_b^A < 0$
Equilibrium B	$G_g^B > 0$	$G_b^B \geq 0$

This and the fact that $G_g = G_b + dD$ (see Eq. 7) implies that $G_{q_1}^C < G_{q_1}^A < G_{q_1}^B$.

Let (π_g^A, π_b^A), (π_g^B, π_b^B) and (π_g^C, π_b^C) denote the strategic regulator's stress testing strategy for the first bank in the equilibria described in Propositions 3, 4 and 5 respectively, assuming the equilibria exist. We can now rank the expected surplus for the strategic regulator in the different equilibria. First, consider

Equilibrium C, in which $\pi_g^C < 1$, $\pi_b^C = 0$ and $G_g^C \leq 0$.

$$V^C(z_1) = \underline{V}(z_1)$$

This follows because in equilibrium, either $G_g^C = 0$, or $G_g^C < 0$ and $\pi_g^* = 0$.

Next, consider Equilibrium A, in which $\pi_g^A = 1$, $\pi_b^A = 0$, and $G_g^A > 0$.

$$V^A(z_1) = \underline{V}(z_1) + \alpha G_g^A$$

Clearly $V^A(z_1) > V^C(z_1)$. Therefore Equilibrium C is strictly dominated by Equilibrium A, whenever they coexist.

Finally, consider Equilibrium B, in which $\pi_g^B = 1$, $\pi_b^B > 0$ and $G_g^B > G_b^B \geq 0$.

$$V^B(z_1) = \underline{V}(z_1) + \left[\alpha G_g^B + (1-\alpha)\pi_b^B G_b^B\right] \geq \underline{V}(z_1) + \alpha G_g^B$$

It follows that $V^B(z_1) > V^0(z_1) + \alpha G_g^A = V^A(z_1)$, because $G_g^B > G_g^A$ as argued above. That is, Equilibrium B strictly dominates Equilibrium A, whenever they coexist. This implies that Equilibrium B also strictly dominates Equilibrium C, whenever they coexist.

7.8 Proof of Proposition 8

Let $V^P(z_1)$ denote the expected surplus for the strategic regulator for the two banks given that the first bank extends a risky loan and let $G_{q_1}^P$ denote the value of G_{q_1} in an economy in which the regulator does not conduct stress tests for the first bank.

We first derive the condition for an uninformative equilibrium to exist. An uninformative equilibrium exists if and only if $G_g > G_b \geq 0$ for $(\pi_g, \pi_b) = (1, 1)$. This is the case if and only if $z_1 < \tilde{z}$ and $\delta \geq \tilde{\delta}_b(z_1)$, where $\tilde{z}$ is defined by Eq. 18 and $\tilde{\delta}_b(z_1)$ is defined by

$$C - dD + \tilde{\delta}_b(z_1)\left[U^*(z_1) - U^*(0)\right] = 0 \tag{28}$$

If an uninformative equilibrium exists, we now show that it dominates any other equilibrium (if exists) for the strategic regulator. $V^P(z_1)$ and $G_{q_1}^P$ coincides with the equilibrium quantities in the uninformative equilibrium. Proposition 7 implies that $V^P(z_1) > V^A(z_1), V^C(z_1)$. We now compare the passive equilibrium to other potential Equilibrium B. In any other Equilibrium B, if exists, $G_b^B = 0 \leq G_b^P$. Therefore $V^P(z_1) \geq V^B(z_1)$ for any other Equilibrium B.